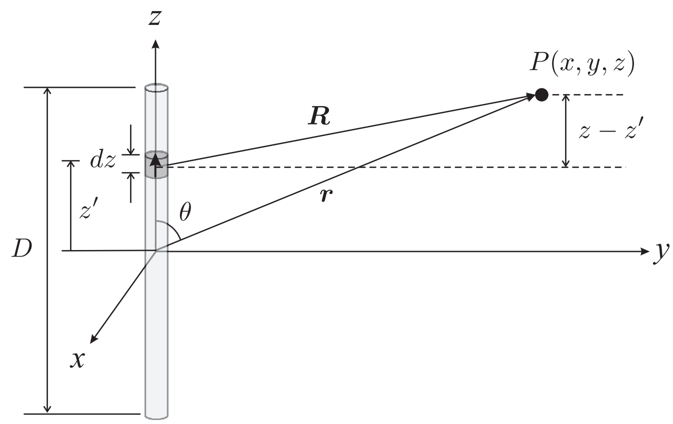
Having just considered the analysis of a half-wave dipole, let us consider for a moment a long wire antenna of length D. The vector potential for a line source directed along the z-axis is found using

$$A_z = \mu \int I(z') \frac{e^{-j\beta R}}{4\pi R} dz'. \tag{1}$$



Recall that we cannot arbitrarily set $R = r$ in the calculation of the potential or fields produced by the antenna, because even for small relative differences between R and r, $\exp(-j\beta r) \neq \exp(-j\beta R)$. In general, one must find the mathematical relationship between r and R to solve the radiation problem.

Let's consider a point P in the yz plane ($x = 0$) for simplicity. From the diagram,

$$R = [y^2 + (z - z')^2]^{1/2} \tag{2}$$
$$= (y^2 + z^2 - 2zz' + z'^2)^{1/2}, \tag{3}$$

and since $r^2 = y^2 + z^2$, and $z = r\cos\theta$,

$$R = (r^2 - 2z'r\cos\theta + z'^2)^{1/2}. \tag{4}$$

Making use of the binomial theorem,

$$(a + b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \ldots, \tag{5}$$

we can write

$$R = r + \frac{1}{2}(r^2)^{-1/2}(-2z'r\cos\theta + z'^2) + \ldots \tag{6}$$
$$\approx r + \frac{1}{2r}(-2z'r\cos\theta + z'^2) \tag{7}$$
$$\approx r - z'\cos\theta + \frac{z'^2}{2r}\sin^2\theta. \tag{8}$$
$$\tag{9}$$

Notice that the approximation becomes exact for $r \to \infty$, leading to the parallel ray approximation we have seen earlier. But for finite r values, the third term (plus the higher order terms we have neglected) for the error between the parallel ray approximation and the actual value for R. This raises the question of exactly when (for what r values) can we invoke the parallel ray approximation? (i.e., what is the minimum value of r that we can use to treat the parallel ray approximation as accurate?)

Technically, this minimum distance or so-called *far field* distance from an antenna is defined when the maximum value of the error term does not exceed $\lambda/16$ (22.5°) at the maximum extent of the antenna $(z' = D/2)$. Since the maximum value of the error term occurs for $\theta = 90^\circ$,

$$\frac{(D/2)^2}{2r_{ff}} = \frac{\lambda}{16} \tag{10}$$

which yields

$$r_{ff} = \frac{2D^2}{\lambda}. \tag{11}$$

This distance is called the far-field distance or *Rayleigh* distance from the antenna. D in general can be taken as the maximum dimension of the antenna, even if it is not a wire antenna. The region

$$\frac{2D^2}{\lambda} < r < \infty \tag{12}$$

is often called the *Fraunhofer region* of the antenna as it is dominated by radiation terms in the antenna fields.

The near-field region of an antenna is commonly subdivided into two subregions:

$$\begin{array}{ll} 0 < r < 0.62\sqrt{D^3/\lambda} & \text{reactive near field} \\ 0.62\sqrt{D^3/\lambda} < r < 2D^2/\lambda & \text{radiating near field} \end{array} \tag{13}$$

The $0.62\sqrt{D^3/\lambda}$ term is found by setting the 4th error term in the Binomial expansion to $\pi/8$.