Plane-Earth Reflection

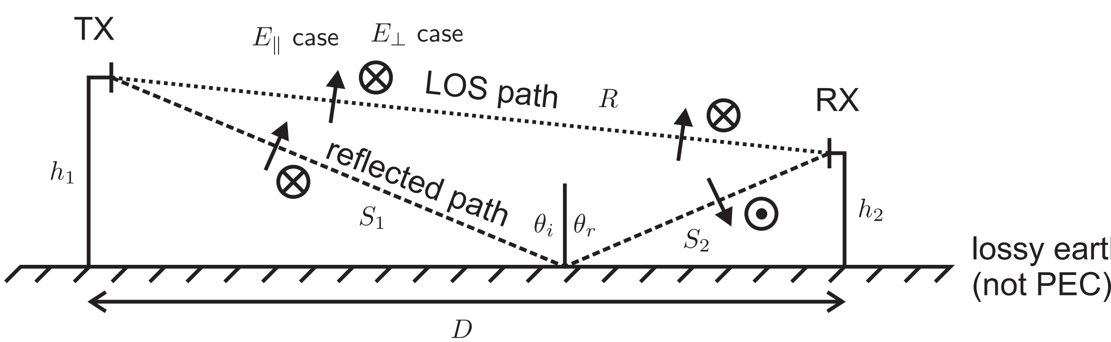
Large-Scale Effects

We are going to consider three major large scale propagation mechanisms in the lectures ahead. These include:

1. *Reflection*, which occurs when a wave impinges on an object with large dimensions relative to a wavelength. The surface of the earth, and large buildings are examples.
2. *Diffraction*, which occurs when an object with sharp edges or irregularities obstructs the radio path between the transmitter and receiver. The edges influence the propagation of the waves around the obstacle.
3. *Refraction*, which occurs when a wave passes through an interface and the angle of the wave vector is changed. We will be most interested in how the atmosphere refracts and bends radio waves, enabling over-the-horizon communications.
4. *Scattering*, which occurs when radio signals are reflected off of objects much smaller than a wavelength, and the density of objects may be fairly high. For example, scattering off of rain drops, which in an aggregate sense leads to large-scale frequency-dependent attenuation of radio signals.

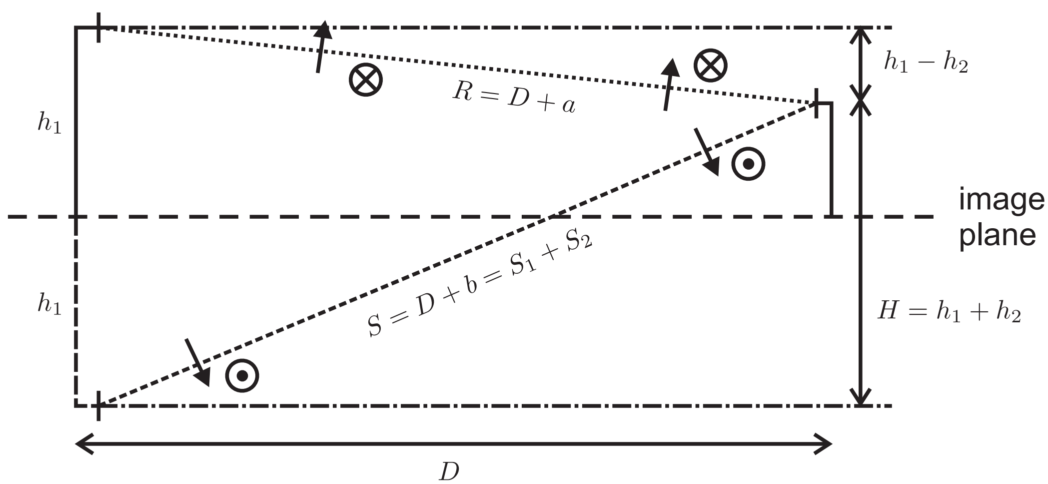
Plane-Earth Reflection

In considering reflections introduced by the environment, the most significant source of reflections in terrestrial links is the earth's surface itself. Consider the link show below:



Sample E-field vectors are shown for the parallel and perpendicular polarization cases. Note that there are two paths from the transmitter (TX) to the receiver (RX): a line-of-sight (LOS) path and a reflected path. The polarizations of the components shown are also consistent with what we have sketched and derived earlier for the case of a TE/TM polarized wave incident upon a media interface. The polarization of the LOS path, regardless of the TE/TM polarization case, remains constant; however, for the reflected path, the polarization of the reflected components are shown 180 degrees out of phase with the LOS path. This results when $\theta_i \to 90^\circ$ (grazing incidence) at an interface with lossy dielectric (not a PEC). The grazing angle assumption is valid since in many cases the antenna heights h_1, h_2 are very small compared to the TX-RX separation. We also assume that the earth is treated as a flat surface for this development, though later on in other problems we will consider a spherical shaped earth surface.

We are interested in the received power at the receiver under these conditions. It is tempting to use image theory to analyze the situation shown. Hote, however, that the "image" in this case produces electric fields that are 180° out of phase with the transmitted field due to the signs of $\Gamma_\parallel$ and $\Gamma_\perp$ for grazing incidence. Hence we invoke a type of "modified" image theory where the vertical components of the field produced by the image are opposite to classical image theory with a PEC ground plane. Alternative, we can think of the boundary conditions being satisfied at the image plane; if we were using PEC-based image theory, the tangential E-fields must sum to zero at the interface, but in the "modified" image theory case, the image plane represents the interface between two dielectrics where the tangential E-fields are *equal*, but do not cancel.



The analysis of this situation parallels the analysis of the 2-dipole array considered some time ago. The two paths to the receiver in this case consist of the LOS path (of length $R = D + a$) and the reflected path (of length $S = D + b$). That is, each path is an additional length a or b longer than the horizontal separation distance D.

Analyzing the larger of the two triangles in the diagram,

$$H^2 + D^2 = S^2 = (D+b)^2 = D^2 + 2Db + b^2. \tag{1}$$

Since $D \ll b$,

$$H^2 \approx 2Db \Rightarrow b = \frac{H^2}{2D} = \frac{(h_1+h_2)^2}{2D}. \tag{2}$$

Similarly, the geometry of the smaller triangle gives

$$a = \frac{(h_1-h_2)^2}{2D}. \tag{3}$$

Using the principle of superposition, the eletric field strength at the receiving antenna is proportional to

$$\frac{e^{-j\beta R}}{R} - \frac{e^{-j\beta S}}{S} = \frac{e^{-j\beta R}}{R}\left[1 - \frac{e^{-j\beta(S-R)}}{S/R}\right]. \tag{4}$$

The term in parentheses is recognized as a correction term for the reflection, a kind of array factor for an antenna and its image, where the image produces a field of *opposite sign* as the line-of-sight path because of 180° phase shift introduced at the reflection interface. The term outside the parentheses is just a standard line-of-sight propagation term that we also see from an antenna.

We can approximate this correction term as follows. First, for the calculation of the exponential term, we can evaluate

$$\begin{aligned} S - R &= (D+b) - (D-a) = b - a \\ &= \frac{(h_1+h_2)^2}{2D} - \frac{(h_1-h_2)^2}{2D} \\ &= \frac{2h_1h_2}{D}. \end{aligned} \tag{5}$$

For the magnitude term,

$$\frac{S}{R} = \frac{D+b}{D+a} = 1 + \frac{b-a}{D} = 1 + \frac{2h_1h_2}{D^2} \approx 1 \tag{6}$$

With these definitions, the field correction term becomes

$$\begin{aligned} 1 - \frac{e^{-j\beta(S-R)}}{S/R} &= 1 - e^{-j\frac{4\pi}{\lambda}\frac{h_1h_2}{D}} \\ &= e^{-j\frac{2\pi}{\lambda}\frac{h_1h_2}{D}}\left(e^{j\frac{2\pi}{\lambda}\frac{h_1h_2}{D}} - e^{-j\frac{2\pi}{\lambda}\frac{h_1h_2}{D}}\right) \\ &= j2e^{-j\frac{2\pi}{\lambda}\frac{h_1h_2}{D}}\sin\left(\frac{2\pi h_1h_2}{\lambda D}\right) \end{aligned} \tag{7}$$

Taking the magnitude of this correction term and squaring, we can apply this to Friis' formula to obtain the received power,

$$\frac{W_r}{W_t} = \frac{G_rG_t\lambda^2}{(4\pi R)^2}\cdot 4\sin^2\left(\frac{2\pi h_1h_2}{\lambda D}\right) \tag{8}$$

$$= \frac{G_rG_t\lambda^2}{(4\pi R)^2}\cdot g_{PE} \tag{9}$$

where g_{PE} is a gain term that accounts for the plane earth reflection (equal to $1/l_{PE}$, the reciprocal of the plane earth reflection loss).

We notice sinusoidal variation as h_1, h_2, λ, and D are varied, which corresponds to an interference process at the receiver. It is possible that the line-of-sight and reflect paths destructively interfere, if they are 180° out of phase, which will lead to complete nulling (fading) of the signal at the receiver. This is obviously a very serious effect to be avoided in any communication system. Furthermore, if we consider what happens at points far away from the receiver such that the argument to the $\sin(\cdot)$ is very small, then

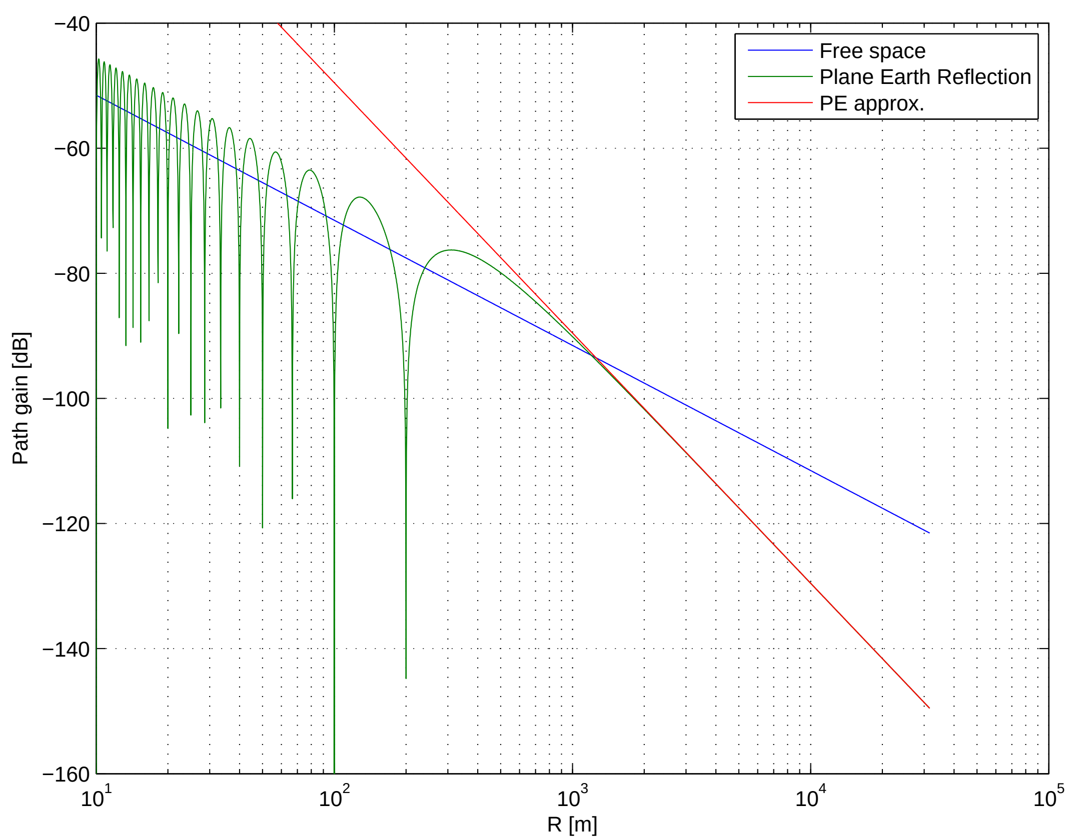
$$\sin^2\left(\frac{2\pi h_1 h_2}{\lambda D}\right) \approx \left(\frac{2\pi h_1 h_2}{\lambda D}\right)^2 \tag{10}$$

and, observing that $D \approx R$,

$$\frac{W_r}{W_t} = \frac{G_r G_t \lambda^2}{(4\pi R)^2} \cdot 4\left(\frac{2\pi h_1 h_2}{\lambda D}\right)^2 = \frac{G_r G_t h_1^2 h_2^2}{R^4}. \tag{11}$$

Hence, two things happen: the received power becomes frequency invariant, and more significantly, the received power decays with $1/R^4$, an incredible rate of decay. Hence, for every 10-fold increase in distance, we lose 40 dB of signal power instead of 20 dB which is the free-space case.

Plots of g_{PE} on a log-log scale are shown on the plot below, where the approximation above is seen to be accurate at large distances. The free-space loss curve is also plotted. The parameters for this plot are $f = 900$ Mhz, $h_1 = 20$ m, $h_2 = 1.667$ m. The fading process is clearly evident for low-moderate R values, with the fading turning into a 40 dB/decade loss pass about $R = 2$ km. Note also that constructive interference yields as much as a 3 dB improvement in the received power at certain points as well.



It is worth noting that we see in free space that the power rolls off according to $1/R^2$ whereas in a worst-case plane-earth case, the power rolls off according to $1/R^4$. In practical scenarios, where there is more than one reflection present, large scale path loss is often modelled according a $1/R^n$ characteristic, where n is called the *path loss exponent* and usually varies between 2 and 4, depending on the propagation environment being considered. A few examples of path loss exponents for different environments are shown in the table below [1].

Environment	Path loss exponent n
Free space	2
Urban area	2.7 to 3.5
Shadowed urban area	3 to 5
In building LOS	1.6 to 1.8
Obstructed in building	4 to 6
Obstructed in factories	2 to 3

References

[1] T. Rappaport, *Wireless Communications: Principles and Practice*, 2nd ed. Prentice-Hall, Dec. 2001.