Field Region Separation

Having just considered the analysis of a half-wave dipole, let us consider for a moment a long wire antenna of length \( D \). The vector potential for a line source directed along the \( z \)-axis is found using

\[
A = \mu \int I(x', y', z') \frac{e^{-jkR}}{4\pi R}dl'.
\] (1)

For the infinitesimal and short dipole, we could approximate \( R \approx r \). However, as the antenna becomes an appreciable fraction (or larger) of a wavelength, phase errors from the \( \exp(\cdot) \) term become significant.

For a dipole on the \( z \)-axis,

\[
R = \sqrt{x^2 + y^2 + (z - z')^2} = \sqrt{x^2 + y^2 + z^2 - 2z'z + z'^2}.
\] (2)

Expressing the field point in spherical coordinates,

\[
R = \sqrt{r^2 - 2rz'\cos \theta + z'^2} = r\sqrt{1 + \frac{(-2rz'\cos \theta + z'^2)}{r^2}}.
\] (3)

Making use of the binomial theorem,

\[
(1 + x)^n = 1 + n.x + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} + \ldots,
\] (4)

we can write

\[
R = r \left\{ 1 - \frac{rz'\cos \theta}{r^2} + \frac{z'^2}{2r^2} - \frac{1}{8} \left( \frac{4r^2z'^2\cos^2 \theta - 4rz'\cos \theta z'^2 + z'^4}{r^4} \right) + \ldots \right\}
\]

\[
= r - rz'\cos \theta + \frac{z'^2}{2r^2} - \frac{z'^2\cos^2 \theta}{2r^2} + \frac{z'^3\cos \theta}{2r^2} - \frac{z'^4}{8r^3} + \frac{1}{16} \frac{(-8r^3z'^3\cos^3 \theta)}{r^5} + \ldots
\]

\[
\approx r - rz'\cos \theta + \frac{z'^2\sin^2 \theta}{2r} + \frac{z'^3\cos \theta}{2r^2} - \frac{z'^4}{8r^3} + \frac{1}{16} \frac{(-8r^3z'^3\cos^3 \theta)}{r^5} + \ldots
\]

(5)

where terms with greater than \( r^{-3} \) dependence are assumed to be negligible.

This completes the derivation of the distance \( R \), which matches equation (4-41) in Balanis. Sections 4.4.1 and 4.4.2 show how the error terms in this equation are constrained to determine the Fraunhofer and Fresnel regions.